SOURCES FOR ELECTROWEAK BARYOGENESIS

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I review a computation of the baryon asymmetry arising from a first order electroweak phase transition in the Minimal Supersymmetric Standard model by classical force mechanism (CFM). I focus on CP violation provided by the charginos and show that it is the usually neglected sum of the two Higgsino fields, $H_1 + H_2$, which gives a larger contribution to the baryon asymmetry than does the combination $H_1 - H_2$. In fact, the latter contribution is exactly zero in CFM, because it is associated with a phase transformation of the fields. Baryogenesis is found to be most effective in MSSM CFM when only $\tilde{t}_R$ is light, which lends independent support for the "light stop scenario", and it remains viable for CP-violating phases as small as $\delta_\mu \sim few \times 10^{-3}$.

1 Introduction

Although CP violation and the phase transition are known to be too weak for baryogenesis within the Standard Model, these problems can be overcome in the Minimal Supersymmetric Standard Model (MSSM). In a small region of MSSM parameter space, corresponding to so called "light stop" scenario[1], the transition may be strong enough to avoid the wash-out of baryon number by sphaleron interactions in the broken phase[1,2]. The sphaleron wash-out computations, while mired with problems associated with the infrared sector of gauge theories, are simple in the sense that one is dealing with equilibrium physics. Situation is markedly different for the theory of baryon production. In this case CP-violating currents are generated inside the bubble walls, diffuse into the plasma in the unbroken phase, and bias sphalerons to produce the baryon asymmetry. By the very axioms of baryogenesis this is an inherently out-of-equilibrium system. As of to date, no theory exists that could tackle the problem in its full extent, while many scenarios have been put forward in an attempt to extract the leading effect in one or the other limit[3,4,5,6]. (However, for an ongoing project with the aim to self-consistently derive the transport equations for baryogenesis see ref.[7].)

Common to all methods is reducing the problem to a set of diffusion equations for the particle species that bias sphalerons. These coupled equations, it is universally agreed, have the general form

$$D_i \mu_i'' + v_w \mu_i' + \Gamma_i(\mu_i + \mu_j + \cdots) = S_i \ , \tag{1}$$

where i labels the particle species, μ_i is its chemical potential, primes denote

spatial derivatives in the direction (z) perpendicular to the wall, v_w is the wall velocity, Γ_i is the rate of an interaction that converts species i into other kinds of particles, and S_i is the source term associated with the current generated at the bubble wall. The essential point, and the one where little agreement exists between different aproaches, is how to properly derive the source terms S_i appearing in (1).

In MSSM, potentially the most dominant source arises from the chargino sector. The CP violating effects are due to the complex parmeters m_2 and μ in the chargino mass term,

$$\bar{\psi}_R M_\chi \psi_L = (\overline{\widetilde{w}^+},\ \overline{\widetilde{h}^+}_2)_R \begin{pmatrix} m_2 & gH_2 \\ gH_1 & \mu \end{pmatrix} \begin{pmatrix} \widetilde{w}^+ \\ \widetilde{h}_1^+ \end{pmatrix}_L . \tag{2}$$

Spatially varying Higgs fields cause the phase of the effective mass eigenstates vary nontrivially over the bubble wall. In all methods that address the thick wall limit[3,4,5,7,9], one computes the current effected by these spatialy varying phases to leading order in an expansion in derivatives of the Higgs fields.

There was an important discrepancy in the literature concerning the derivative expansion of the chargino source. References[8] and [11] obtained a source for the $H_1 - H_2$ combination of higgsino currents of the form

$$S_{H_1-H_2} \sim \mathrm{Im}(m_2\,\mu)\,(H_1 H_2' - H_2 H_1'), \tag{3}$$

whereas ref.[9], albeit unknowingly, found the other orthogonal linear combination, $H_1 + H_2$, for which the result is

$$S_{H_1+H_2} \sim \mathrm{Im}(m_2\,\mu)\,(H_1 H_2' + H_2 H_1'), \tag{4}$$

We have recently understood[10] that this disagreement about the sign is spurious and that all three methods actually agree with eq. (4); it simply was not computed by the other authors of the references[8,11,5].

The reason that the combination $H_1 + H_2$ was not considered by the other authors is because it tends to be suppressed by Yukawa interactions and helicity-flipping interactions from the μ term in the chargino mass matrix. Indeed, if all the interactions arising from the Lagrangian

$$\begin{aligned} V = y\mu\tilde{h}_1\tilde{h}_2 + h_2\bar{u}_R q_L + y\bar{u}_R\tilde{h}_{2L}\tilde{q}_L + y\tilde{u}_R^*\tilde{h}_{2L}q_L \\ - y\mu h_1\tilde{q}_L^*\tilde{u}_R + yA_t\tilde{q}_L h_2\tilde{u}_R^* + \mathrm{h.c.}, \end{aligned} \tag{5}$$

are considered to be in thermal equilibrium, they give rise to the constraints $\xi_{H_1} - \xi_{Q_3} + \xi_T = 0$ and $\xi_{H_2} + \xi_{Q_3} - \xi_T = 0$, which would damp out the effect of

the source $S_{H_1+H_2}$. The rates Γ_A of the processes coming from (5) are finite however, so the equilibrium relations are satisfied only up to corrections of order $(D_i\Gamma_A)^{-1/2}$, where D_i is the diffusion coefficient for Higgs particles or quarks. Using the Higgs diffusion constant $D_h \sim 20/T$ and the Yukawa rate $\Gamma \sim 3y^2T/16\pi$[9], one finds only a mild suppression factor $(D_h\Gamma)^{-1/2} \sim 1$. The source $S_{H_1-H_2}$ on the other hand suffers from a serious suppression: baryon number generated is (obviously) proportional to a spatial variation of H_2/H_1, but relative deviations from constancy of this ratio have been found to be very small, in the range $10^{-2} - 10^{-3}$. [12,13] Therefore the source $S_{H_1-H_2}$ should be expected to be subdominant to $S_{H_1+H_2}$ even in the models of refs. [11,12]. In the CFM the situation is even worse, because there the source for $S_{H_1-H_2}$ actually vanishes, as we shall see below.

2 Semiclassical Boltzman equation

The classical force baryogenesis rests on particularily appealing intuitive picture. One assumes that the plasma in the condensate region can be described by a collection of semiclassical WKB-states, following world lines set by their WKB-dispersion relations and corresponding canonical equations of motion. One can then immediately write down a semiclassical Boltzman equation for the transport

$$(\partial_t + \mathbf{v}_g \cdot \partial_{\mathbf{x}} + \mathbf{F} \cdot \partial_{\mathbf{p}}) f_i = C[f_i, f_j, ...]. \tag{6}$$

where the group velocity and the classical force are given by

$$\mathbf{v}_g \equiv \partial_{\mathbf{p}_c}\omega \qquad \mathbf{F} = \dot{\mathbf{p}} = \omega\dot{\mathbf{v}}_g, \tag{7}$$

where $\mathbf{p}_c$ is the canonical and $\mathbf{p} \equiv \omega\mathbf{v}_g$ is the physical, kinetic momentum along the WKB-world line. Because of CP-violating effects particles and antipartices experience different force in the wall region, $F_{\rm ap} \neq F_{\rm p}$, which leads to separation of chiral currents. What remains is to compute the disperson relation to obtain the group velocity and the force, after which the diffusion equations follow from (6) in a standard way by truncated moment expansion[9].

2.1 Dispersion relation

I will first consider the example of a single Dirac fermion with a spatially varying complex mass:

$$(i\gamma^\mu\partial_\mu - mP_R - m^*P_L)\psi = 0; \qquad m = |m(z)|e^{i\theta(z)}, \tag{8}$$

where $P_{L,R} = (1 \mp \gamma_5)/2$. Assuming planar walls I will also boost to the frame in which the momentum parallel to the wall is zero, $p_x = p_y = 0$ (I am ignoring

the effects of thermal background here). In this simple case it is fairly easy to solve the whole wave function to the first nontrivial order in the gradients,

$$\psi_s = \frac{|m|}{\sqrt{2p_s^+(\omega + sp_0)}} \begin{pmatrix} 1 \\ \frac{\omega + sp_s^+}{|m|} \end{pmatrix} \chi_s \, e^{i\int \tilde{p}_s + i\frac{\theta}{2}\gamma_5 + i\phi_G}, \tag{9}$$

where $p_0 \equiv \sqrt{\omega^2 + m^2}$, $\tilde{p}_s \equiv p_0 + s\omega\theta'/(2p_0)$, $p_s^+ \equiv \tilde{p}_s + \omega\theta'/2$, with $\theta' \equiv \partial_z \theta$, and $\sigma_3 \chi_s = s\chi_s$. The phase of the wave function in (9) can be written as an integral over the local (canonical) momentum:

$$p_c = p_0 + \frac{s\theta'}{2p_0}(\omega \pm sp_0) + \alpha_G'. \tag{10}$$

This is, of course, just the usual WKB-dispersion relation which has been derived in many places[4,9]. The presence of an arbitrary function α_G'[a] shows explicitly, as one should expect, that p_c is a gauge dependent quantity. The physical quantities are gauge independent, however. For example, in the computation of the group velocity, the gauge dependent parts (including the chiral rotation proportional to $\pm\theta'$) vanish because they are ω-independent:

$$v_g = \partial_{p_c}\omega = (\partial_\omega p_c)^{-1} = \frac{p_0}{\omega}\big(1 + \frac{sm^2\theta'}{2p_0^2\omega}\big) \tag{11}$$

Similar equation holds for antiparticles, but with $\theta \to -\theta$. The gauge independency of the current $j^\mu = \bar{\psi}\gamma^\mu\psi$ is obvious from (9). Moreover, it is easy to show by direct substitution that

$$j^\mu = (1/v_g\,; \hat{\mathbf{p}}). \tag{12}$$

Thus, in the absence of collisions, the WKB-particles merely follow their trajectories (corresponding to the stationary phase of the wave) and if they slow down at some point, the outcome is an increase of local density. The crux of the CFM is that where particles slow down, antiparticles speed up in relation, leading to a local particle-antiparticle bias.

2.2 Physical force

We still need to see how the classical force arises from the dispersion relation. Physically, one expects that force simply corresponds to acceleration, as was

[a]It may be introduced at any point by a local phase transition $\psi \to e^{i\alpha_G(x)}\psi$, which leaves the lagrangian invariant.

assumed above in Eq. (7). It is instructive to see that this force is consistent with the canonical equations of motion. First note that the physical momentum $p \equiv \omega v_g$, may be written in terms of canonical momentum as

$$p \;\simeq\; p_c^{\pm} - \alpha^{\pm} - \frac{s\theta' p}{2\omega}. \tag{13}$$

where $\alpha^{\pm} = \alpha'_G \pm \theta'/2$. Force acting on this momentum is then

$$F = \dot{p} = \dot{p}_c - \dot{z}\partial_z(\alpha^{\pm} + \frac{s\theta' p_k}{2\omega}). \tag{14}$$

Using the canonical equations $\dot{z} = v_g$ and $\dot{p}_c = -(\partial_z \omega)_{p_c}$, along with the energy conservation, one finds that

$$F = -\frac{mm'}{\omega} + \frac{s(m^2\theta')'}{2\omega^2} = \omega v_g \partial_z v_g = \omega \dot{v}_g, \tag{15}$$

in accordance with (7). Note that while the *canonical force* $F_c \equiv -(\partial_z \omega)_{p_c}$ is obviously gauge dependent, the gauge parts cancel in the expression for the physical force F. Again, for antiparticles $\theta \to -\theta$, so that the second term in (15) is the CP-violating force, which leads to baryon production.

3 Baryogensis from chargino transport

The WKB-analysis of the chargino sector proceeds very similarly to the above simple example. Naturally there are some complications due to the additional 2×2 flavour mixing structure. After a little algebra one finds the dispersion relation

$$\begin{aligned} p_{H_{i\pm}} \;&=\; p_{0\pm} \mp \frac{s(\omega + sp_{0\pm})}{2p_{0\pm}} \frac{\Im(m_2\mu)}{m_\pm^2 \Lambda}(u_1 u_2' + u_2 u_1') \\ &\qquad \mp s_{H_i} \frac{2\Im(m_2\mu)}{\Lambda + \Delta}(u_1 u_2' - u_2 u_1') + i\alpha'_{i\pm} \end{aligned} \tag{16}$$

where $u_i \equiv gH_i$, $\Lambda = m_+^2 - m_-^2$ and $\Delta = |m_2|^2 - |\mu|^2 + u_2^2 - u_1^2$. If $m_2 > \mu$, ($m_2 < \mu$) then the larger (smaller) mass eigenstate m_+ (m_-) corresponds to higgsinos. Although promisingly $s_{H_1} = -s_{H_2} = 1$, the $(u_1 u_2' - u_2 u_1')$-term does not source the combination $H_1 - H_2$, because it vanishes when differentiated with respect to ω. (It could also be absorbed into the arbitrary phase functions $\alpha_{i\pm}$ arising from freedom to perform field redefinitions.) Apart from this "gauge" phase, both higgsinos have identical dispersion relations and

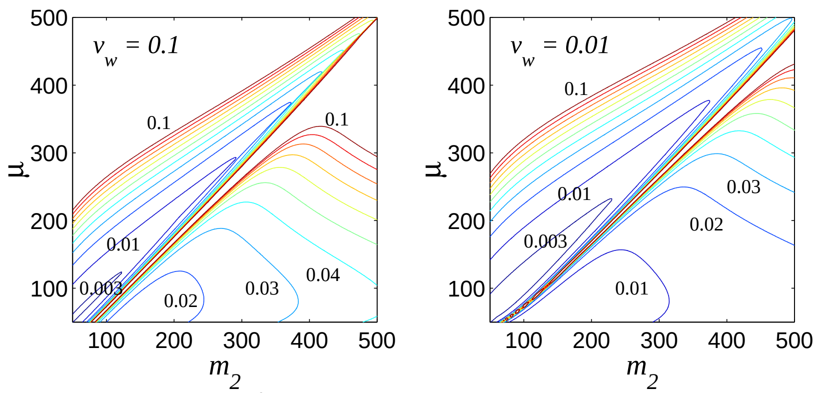

Figure 1: The contours of δ_μ which give rise to baryon asymmetry of $\eta_B = 3 \times 10^{-10}$.

hence have identical sources in their diffusion equations, from which it follows that $S_{H_1-H_2} = 0$ in CFM. The nonvanishing source has a very simple form[10]

$$S_{H_1+H_2} = -\frac{s}{2}\frac{v_w D_h}{\langle p^2/\omega^2\rangle_\pm}\langle p_z/\omega^3\rangle_\pm \big(m_\pm^2\theta_{\rm e}'\big)'', \tag{17}$$

where $\langle\cdots\rangle$ refers to thermal average and $m_\pm^2\theta_{\rm e}' \equiv \Im(m_2\mu)(u_1u_2' + u_2u_1')/\Lambda$. The appropriate diffusion equations have been set up and solved in reference[10]. The final baryon number can be written as a one-dimensional integral over the source

$$\eta_B \propto \frac{\Gamma_{\rm sph}}{v_w}\, C_{sq} \int_{-\infty}^{\infty} dz S_{H_1+H_2}(z)\mathcal{G}(z), \tag{18}$$

where $\Gamma_{\rm sph}$ is the Chern-Simons number diffusion rate in the symmetric phase[14], v_w is the wall velocity and $\mathcal{G}(y)$ is a Greens function which I do not write explicitly here[10]. The parameter C_{sq} encodes the essential squark spectrum dependence of our results: if only $\tilde t_R$ is light then $C_{sq} = 5/23$. If, in addition, $\tilde t_L$ and $\tilde b_L$ are light then $C_{sq} = 1/41$ and finally, if $\tilde t_L$, $\tilde b_L$ and $\tilde b_R$, and any number of other squarks are light then $C_{sq} = 0$. This trend lends striking and entirely independent support for the wash-out motivated light stop scenario[1]. In Fig. 1 shown are the contours of $\delta_\mu = arg(\mu)$ corresponding to the eventual baryon to photon ratio of $\eta_B = 3 \times 10^{-10}$ for $v_w = 0.1$ and $v_w = 0.01$. Baryogenesis is seen to remain viable in the MSSM at least for δ_μ as small as *few* $\times 10^{-3}$.

4 Conclusions

I have reviewed baryogenesis via the classical force mechanism (CFM) from the chargino transport in the Minimal Supersymmetric Standard Model. It was shown that the physical quantities entering the CFM computation are unambiguos and independent of phase transformations on fields. It was pointed out that the dominant source for baryogenesis in the thick wall limit is the one corresponding to the linear combination of higgsinos $H_1 + H_2$, despite the suppression by top-Yukawa strength interactions, because the corresponding suppression is much milder than the suppression on $H_1 - H_2$ arising due to need for non-constancy of H_2/H_1 over the bubble wall[11,12]. I suggest that this linear combination should lead to dominant effect also in the thin wall limit[6]. It was also observed that CFM is most efficient for the case when as few squarks as possible are light, which lends support for the so called "light stop scenario"[1], necessary for avoiding the baryon wash-out in the broken phase. It was finally shown that the CFM may be able to produce the observed baryon asymmetry with the explicit CP-violating phase δ_μ well below present observational limits.

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